Infrastructure Investment Decisions in Multimodal Intercity Transportation Networks: An Equilibrium Approach

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Abstract
Given the huge amount of cost involved with infrastructure capacity investment, which is an important means to mitigate congestion, justifying the benefit returns is of critical importance when making investment decisions. To this end, one should evaluate the response of transportation system to investment. Compared to the previous studies that performed partial equilibrium analyses by considering either private (road) or public (air) transportation systems separately, this paper aims at analyzing general equilibrium where private and public transportation are modeled in one single framework. This novel framework explicitly captures decisions of all system components by modeling fare and service frequency competition between airlines, and travelers’ joint mode-route choices. The interaction between system components results in a multi-leader-follower game, which is formulated as an equilibrium problem with equilibrium constraints (EPEC). The proposed EPEC is solved by combining Gauss-Seidel diagonalization method and relaxation method and is implemented on a test network.

Keywords: equilibrium, air, road, multi-leader-follower game, EPEC.

1. Introduction
This paper studies the problem of infrastructure investment decision making in intercity multimodal (air and road) transportation systems. Infrastructure capacity investment is recognized as an important means to mitigate congestion in the transportation system. To make the best returns to this huge amount of financial burden on the public, policy-makers need to know the response of transportation system to investment. To this end, this paper proposes an equilibrium-based benefit assessment framework. Since a multimodal intercity transportation system is composed of a heterogeneous set of stakeholders including suppliers (i.e., airlines/carriers who provide public transportation services) and users (i.e., individual travelers), the proposed framework explicitly takes into account all suppliers’ and users’ decision making and their interaction, while also incorporating system performance in terms of congestion delay.

Equilibrium modeling in intercity transportation has garnered increasing attentions in the research community, which can be categorized in two classes: 1) analytical models, which typically model suppliers’ competition over small networks, such that closed-form equilibrium solutions can be obtained (e.g., Noruzoliaee et al., 2015; Silva and Verhoef, 2013; Zhang and Wei, 1993; Zhang and Czerny, 2012); and 2) operations research based models (e.g., Adler, 2005; Adler et al., 2010; Adler et al., 2014; Hansen and Kanañani, 1990; Hong and Harker, 1992; Li et al., 2010), in which the presence of multiple competing carriers is typically modeled as an n-player noncooperative Nash game. The game is then formulated as simultaneous profit maximization problems of carriers, subject to user equilibrium conditions. The linkages between the carriers’ problems can occur in the user constraints and system performance measures through congestion effects.

Previous studies have focused on partial equilibrium analyses by performing separate analyses on private (road) and public (air) transportation systems. Since there are no carriers involved in road transportation systems, the objective of the first group of studies (e.g., Fisk, 1980; Sheffi, 1985) is to model the interaction between road travelers by also accounting for system performance in terms of congestion, which will result in a performance-demand equilibrium analysis. The second group of studies focuses on public transportation, where carriers provide services to travelers, and probe into supply-demand equilibrium by explicitly accounting for the competition between carriers, travelers’ decisions, and the interaction between carriers and travelers. The objective of this paper is to fill a major gap in the literature, by performing a general equilibrium analysis in that air-road competition is explicitly taken into consideration. This holistic approach

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leads to an integrated supply-demand-performance equilibrium where none of the carriers and travelers can unilaterally change its decision to better off. To this end, we model the interaction of all system components as a novel multi-leader-follower game, which is formulated as an equilibrium problem with equilibrium constraints (EPEC).

The remainder of this paper is organized as follows. After specifying equilibrium components in section 2, we characterize intercity multimodal transportation system equilibrium as an integrated supply-demand-performance equilibrium formulation in section 3. Infrastructure investment decision making is then explored in section 4. Solution algorithm is illustrated in section 5 followed by a numerical analysis in section 6. The paper concludes in section 7.

2. Equilibrium components

The preliminary step in formulating system equilibrium is to specify equilibrium components. On the supply side, we present the structure of operating costs. On the demand side, we portray the travelers’ behavioral model by also accounting for system operational performance. To facilitate the exposition this paper employs the notation in Table 1.

A multimodal intercity transportation network (graph) $G$ is modeled as a multi-layer network composed of many nodes $N$ (e.g., airports and road intersections), which are connected to each other by a set of arcs $A$ (e.g., flight segments and road links). Each traveler makes his/her trip between an OD pair (market) through a sequence of arcs (i.e., route $p \in P$) by choosing a carrier $k \in K$.

Table 1. Notation

<table>
<thead>
<tr>
<th>Network</th>
<th>$G(N, A)$</th>
<th>multimodal transportation network</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>set of nodes in network ($n \in N$)</td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>set of arcs (links) in network ($a \in A$)</td>
<td></td>
</tr>
<tr>
<td>$W$</td>
<td>set of origin-destination (OD) pairs on network ($w \in W$)</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>set of routes (paths) between all OD pairs ($p \in P$)</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>set of carriers ($k \in K$)</td>
<td></td>
</tr>
<tr>
<td>$PK$</td>
<td>set of route-carrier mixes (${p, k} \in PK$)</td>
<td></td>
</tr>
<tr>
<td>$\delta_{n,a}$</td>
<td>1 if arc $a \in A$ is connected to node $n$; 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>$\delta_{a,(p,k)}$</td>
<td>1 if arc $a \in A$ is on route-carrier mix $(p,k)$; 0 otherwise</td>
<td></td>
</tr>
<tr>
<td>$\kappa_{a}(k_n)$</td>
<td>capacity of arc $a$ (node $n$)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Demand</th>
<th>$q_{sw}$</th>
<th>maximum number of potential passenger trips between OD pair $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q_{w}$</td>
<td>total passenger trips between OD pair $w$</td>
</tr>
<tr>
<td></td>
<td>$q_{p,k}$</td>
<td>passenger traffic on route $p$ by carrier $k$ between OD pair $w$</td>
</tr>
<tr>
<td></td>
<td>$V_{p,k}^w$</td>
<td>utility of a traveler by choosing a route-carrier mix $(p,k)$ on OD pair $w$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Supply/Performance</th>
<th>$R_{p,w}$</th>
<th>fare (ticket price) on route $p$ charged by carrier $k$ between OD pair $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F_{a,k}^w$</td>
<td>frequency of service on arc $a$ provided by carrier $k$</td>
</tr>
<tr>
<td></td>
<td>$S_{a,k}^w$</td>
<td>aircraft size (seat capacity) on arc $a$ provided by carrier $k$</td>
</tr>
<tr>
<td></td>
<td>$C_{a,k}^w$</td>
<td>(constant) unit operating cost on arc $a$ incurred by carrier $k$</td>
</tr>
<tr>
<td></td>
<td>$C_{p,k}^w$</td>
<td>(constant) unit operating cost on route-carrier mix $(p,k)$ on OD pair $w$</td>
</tr>
<tr>
<td></td>
<td>$SD_{p,k}^w$</td>
<td>schedule delay on route $p$ by carrier $k$ between OD pair $w$</td>
</tr>
<tr>
<td></td>
<td>$ST_{p,k}^w$</td>
<td>scheduled travel time on route $p$ by carrier $k$ between OD pair $w$</td>
</tr>
<tr>
<td></td>
<td>$CD_{p,k}^w$</td>
<td>congestion delay on route $p$ by carrier $k$ between OD pair $w$</td>
</tr>
<tr>
<td></td>
<td>$VC_{a}(VC_{a,n})$</td>
<td>volume over capacity ratio of arc $a$ (node $n$)</td>
</tr>
</tbody>
</table>

2.1. Supply: operating costs

Ticket price (fare) and service frequency are two key supply factors offered by carriers to travelers. In this paper, these supply factors are endogenously determined by explicitly considering the competition between carriers. Carriers incur costs to provide service, which will affect the offered service price and frequency. This sub-Section specifies these operating costs.
A significant portion of total costs incurred by air carriers is attributed to aircraft-related costs, namely operating costs. Operating costs include fixed cost (i.e., aircraft ownership cost) and variable costs (i.e., fuel cost, pilots and cabin crew costs, airport fees, air traffic control fee, aircraft maintenance cost, and aircraft insurance premium). An air carrier’s operating costs are characterized on a per aircraft-trip basis. Specifically, we use the empirical results from (Swan and Adler, 2006) that aircraft operating costs on flying link (i.e., flight segment) \( a \) by carrier \( k \), \( C^k_a \), is a linear function of trip length \( L_a \) (measured in miles) and the seating capacity of the aircraft (i.e., aircraft size) chosen by carrier \( k \) on link \( a, S^k_a \).

\[
C^k_a = 0.0306(L_a + 448)(S^k_a + 104) \quad \forall a \in A; \ k \in K^{air} \tag{1}
\]

Road users, on the other hand, will incur operating cost by opting for traveling with their own vehicle. According to U.S. Bureau of Transportation Statistics (2014) and assuming the average vehicle occupancy of 1.5 passengers/vehicle following Levinson and Gillen (1998), in this study we consider the following formula for the automobile operating cost per vehicle-trip over road arc \( a \):

\[
C^k_a = 0.19L_a \quad \forall a \in A; \ k \in K^{road} \tag{2}
\]

### 2.2. Performance: congestion delay

Congestion delay is a key factor affecting decisions of carriers and travelers by imposing additional costs to them and degrading level of service, the effect of which is more pronounced when transportation network is highly loaded. In light of this, in this subsection we specify congestion delay functions for air and road networks separately.

Air travel congestion, measured as the average airport arrival delay at the airport level, is specified as a function of the ratio of total flight arrivals to runway arrival capacity, as shown in Equations (3.1)-(3.2). We do not consider departure delay at an airport, in that departure delay can be absorbed en route by flight schedule buffer (Zou and Hansen, 2014). Empirical evidence also suggests that passenger demand is not sensitive to flight departure delays (Hsiao and Hansen, 2011). Statistical models for airport arrival delay have been estimated by various researchers, with recent ones including Liu et al. (2013), Swaroop et al. (2012), and Zou (2012). Equation (3.2), which is inspired by Zou (2012), specifies congestion delay at a node (i.e., airport) of air transportation network, i.e., \( C_D_n \). Congestion delay on a flight segment, which is equal to congestion delay at destination airport, and congestion delay on a route, which is the sum of congestion delays on all arcs on the route, can then be determined by Equations (3.3)-(3.4), respectively.

\[
V_C_n = \sum_{k \in K^{air}} \frac{\delta_{n,a} \rho_n^k}{\kappa_n} \quad \forall n \in N^{air} \tag{3.1}
\]

\[
C_D_n = \delta_0 + \delta_1 V_C_n + \delta_2 V_C_n^2 \quad \forall n \in N^{air} \tag{3.2}
\]

\[
C_D_a = \sum_{n \in N^{air}} \delta_{n,a} C_D_n \quad \forall a \in A^{air} \tag{3.3}
\]

\[
C_D_{p,k}^w = \sum_{a \in A^{air}} \delta_{a(p,k)} C_D_a \quad \forall (p, k) \in PK_w | k = \text{air}, w \in W^k \tag{3.4}
\]

where \( \delta_0, \delta_1, \) and \( \delta_2 \) are model parameters.

In contrast to air transportation with congestion concentrated in network nodes, congestion delay takes place along network links in road networks. According to the U.S. Bureau of Public Roads (BPR), congestion delay experienced by road users on a link \( a \), i.e., \( C_D_a \), is a function of the ratio of the link traffic volume, which is determined by Equation (4.1), to its capacity, as shown in Equations (4.2)-(4.3), where \( \alpha \) and \( \beta \) are model parameters and assuming average vehicle occupancy of 1.5 passengers/vehicle following Levinson and Gillen (1998). Congestion delay on a route can be simply computed by summing congestion delays on all arcs on the route, as shown in Equation (4.4).

\[
v_a = \sum_{k \in K} \sum_{w \in W} \sum_{(p,k) \in PK_w \mid k} \delta_{a(p,k)} q_{p,k}^w \quad \forall a \in A; \ (p, k) \in PK_w \mid k = \text{road}, w \in W^k \tag{4.1}
\]
\[ VC_a = \frac{v_a}{1.5\kappa_a} \quad \forall a \in A^{road} \]  
\[ CD_a = ST_a \alpha VC_a^{\beta} \quad \forall a \in A^{road} \]  
\[ CD^w_{p,k} = \sum_{a \in A^{road}} \delta_{a,(p,k)} CD_a \quad \forall (p,k) \in PK_w | k = \text{road}, w \in W^{k} \]

2.3. Demand: joint mode-route decisions

This research considers a two-level Nested Logit aggregate demand model that simultaneously copes with demand generation and assignment for each OD pair (Figure 1). At the upper level which is about demand generation, we assume that the OD pair of interest has a maximum number of potential trips, termed saturated demand \( q_{w}^{sat} \), which could be made. The saturated demand is determined by the socioeconomic and geographic characteristics of the OD pair (e.g., population and per capita income level at the two ending cities, and the distance of the OD pair). Realized demand \( q_{w} \), however, is often a very small portion of the saturated demand.

Given that the amount of realized travel demand is generated for an OD pair, at the lower level of the NL model travelers are assigned to different route-carrier combination options that are available for the OD pair. The choice of a representative individual is assumed to maximize one’s travel utility, which is a function of travel characteristics that are perceivable of the corresponding route-carrier mix, as well as an unobserved component.

![Figure 1. Nesting structure of the demand model (for OD pair w)](image)

One of the most perceivable travel characteristics for each route-carrier choice is normal trip time (for air travel, this means scheduled transit time; for road travel, this means unimpeded travel time). When air is involved, supply characteristics such as fare and service frequency are also important. We further incorporate performance (in terms of congestion delay) into the utility function since congestion affects actual travel time and thus travel utility.\(^2\) This research considers that congestion mainly occurs to road segments and airports.

Following the above discussion, the utility of a representative traveler facing a route-carrier mix \((p,k)\) on OD pair \( w \), \( U^w_{p,k} \), is specified as follows. In (5.1), \( V^w_{p,k} \) denotes the perceived utility which is a linear function of travel characteristics as in (5.2)-(5.3). \( \epsilon^w_{p,k} \) represents the unobserved component (error term).

\[ U^w_{p,k} = V^w_{p,k} + \epsilon^w_{p,k} \quad \forall (p,k) \in PK_w, w \in W^{k} \]  
\[ V^w_{p,k} = \gamma_k + \gamma_{R}C^w_{p,k} + \gamma_{ST}ST^w_{p,k} + \gamma_{CD}CD^w_{p,k} \quad \forall (p,k) \in PK_w | k = \text{road}, w \in W^{k} \]  
\[ V^w_{p,k} = \gamma_k + \gamma_{R}R^w_{p,k} + \gamma_{ST}ST^w_{p,k} + \gamma_{CD}CD^w_{p,k} + \gamma_{SD}SD^w_{p,k} \quad \forall (p,k) \in PK_w | k = \text{air}, w \in W^{k} \]

\(^2\)To be sure, travelers will not be able to know the actual trip time when making travel decisions. The congestion effect here should be interpreted as travelers’ perception of past congestion using the same route.
where \( Y_{fk}, Y_{SD}, Y_{ST}, Y_{CD} \) are negative parameters since an increase in each of fare, schedule delay, unimpeded (scheduled) travel time, and congestion delay will not benefit travelers, thereby decreasing their utilities. \( Y_{ks} \) is a carrier-specific parameter, which can be attributed to service quality offered by a carrier. \( C_{pk}^{w} \) is the total automobile operating cost on route \( p \), which is the sum of operating costs on all connecting links, i.e., \( C_{pk}^{w} = \sum_{a \in A(k)} (\delta_{a,\{p,k\}} C_{a}) \); \( S_{pk}^{w} \) is the scheduled (or unimpeded) travel time on route-carrier mix \((p,k)\) that is a function of distance traveled and average speed; \( C_{DP_{pk}^{w}} \) is congestion delay on route \( p \), which is specified in Equations (3.4) and (4.4) for air and road networks, respectively. \( R_{w}^{p,k} \) is the endogenous fare determined by carrier \( k \) for traveling on route \( p \); and \( SD_{k}^{w} \) is the average schedule delay (Li et al., 2010) on route-carrier mix \((p,k)\) that, according to Equation (6.2), is a function of endogenous service frequencies offered by the carrier on all arcs on the route, as shown in Equation (6.1);

\[
SD_{a}^{k} = \frac{\delta_{a}^{k}}{4T_{a}} \quad \forall a \in A; k \in K^{air} \tag{6.1}
\]

\[
SD_{p,k}^{w} = \sum_{a \in A^{k}} \delta_{a,\{p,k\}} SD_{a}^{k} \quad \forall (p,k) \in PK_{w} | k = \text{air}, w \in W^{k} \tag{6.2}
\]

Under the assumption that \( \omega_{p,k}^{w} \) are identically and independently Gumbel distributed, the market share of route-carrier mix \((p,k) \in PK_{w} \) in the realized demand for OD pair \( w \) has a simple Multinomial Logit form:

\[
MS_{(p,k)|w} = \frac{\exp\left(\frac{1}{\theta} V_{p,k}^{w}\right)}{\sum_{(i,j) \in PK_{w}} \exp\left(\frac{1}{\theta} V_{i,j}^{w}\right)} \quad \forall (p,k) \in PK_{w}, w \in W^{k} \tag{7}
\]

Returning to the upper level, the market share of realized demand in total saturated demand depends on the inclusive value (i.e., logsum) of all route-carrier mixes. Note that the utility of the no-travel option in Equation (8.1) is normalized to zero.

\[
MS_{w} = \frac{\exp(\theta l_{w})}{1 + \exp(\theta l_{w})} \quad \forall w \in W \tag{8.1}
\]

\[
l_{w} = \ln \left[ \sum_{(i,j) \in PK_{w}} \exp \left( \frac{1}{\theta} V_{i,j}^{w} \right) \right] \quad \forall w \in W \tag{8.2}
\]

where \( 0 < \theta < 1 \) captures the scale effects associated with nesting. Note that Equation (8.1) implies an elastic demand function where the realized OD demand is a function of the level of service, which is represented by the logsum term, i.e., \( q_{w} = Q_{w}(\theta l_{w}) \).

Combining Equations (7) and (8.1) and assuming that saturated demand for OD pair \( w \) is known as \( q_{w}^{sat} \), travel demand for carrier-route combination \((p,k) \in PK_{w} \) can be calculated using the following formula:

\[
q_{p,k}^{w} = q_{w}^{sat} \cdot MS_{w} \cdot MS_{(p,k)|w} = q_{w}^{sat} \cdot \frac{\exp(\theta l_{w})}{1 + \exp(\theta l_{w})} \cdot \frac{\exp\left(\frac{1}{\theta} V_{p,k}^{w}\right)}{\sum_{(i,j) \in PK_{w}} \exp\left(\frac{1}{\theta} V_{i,j}^{w}\right)} \quad \forall (p,k) \in PK_{w}, w \in W^{k} \tag{9}
\]

3. Multimodal transportation system equilibrium

In this section we introduce a new mathematical framework to characterize intercity multimodal transportation system equilibrium, which can be later exploited as a crucial tool to assess benefit returns of infrastructure investment decisions. The discussion of system equilibrium in this paper takes into account all suppliers’ (i.e., carriers’) and travelers’ decision making and their interaction. We first establish the competition between carriers (supply side) as a generalized Nash Equilibrium (GNE) problem and formulate it as a set of non-linear mathematical programs. Then we consider the interaction of travelers with each other (demand side) by means of the stochastic user equilibrium (SUE) problem with elastic demand and establish its equivalent variational inequality (VI) problem. Finally, we probe into the interaction between carriers and travelers by integrating supply and demand, which results in a multi-leader-follower game shown in Figure 2, where carriers act as leaders who are capable of anticipating followers’ (travelers’) decisions in response to
their own strategic decisions. Followers, on the other hand, will observe carriers’ strategic decisions of fare and service frequency and decide whether or not to travel and choose their travel mode and route.

![Diagram of Multi-leader-follower game](image)

**Figure 2. Multi-leader-follower game**

### 3.1. Supply: fare and frequency competition

The multimodal intercity transportation system excluding roads is characterized by an oligopolistic market where a handful of airlines compete for passengers, by adjusting their supply such as fare and service frequency. The competition among the carriers can be naturally formulated as an n-player, non-cooperative Nash game, in which the objective of each independent player (i.e., carrier) is to maximize its profit (or other objective) across its network. Assuming that a carrier is a profit maximizer, the profit is the difference of the carrier’s total revenue and total cost. Total revenue of a carrier is the total ticket fares raised across the carrier’s network. Total cost is the sum of three terms: 1) operating cost, which accounts for aircraft-related costs; 2) congestion delay cost, which explicitly incorporates additional costs incurred by carriers due to congestion delay (e.g., more fuel burn, crew cost, etc.); and 3) passenger-related costs, which correspond to different passenger-related costs (e.g., airport passenger transfer and non-transfer charges, security screening costs, baggage handling, food onboard, some part of ticketing expenses and commissions, etc.).

The profit maximization problem of each carrier $k \in K$ is shown in (10.1)-(10.5). When optimizing its objective, a Nash player treats the competing carriers’ decisions ($X^{-k}$) as constant.

$$\begin{align*}
\text{Max } \pi^k(X^k; X^{-k}) &= \sum_{w \in W} \sum_{(p,k) \in PK_w} R^w_{p,k} \cdot q^w_{p,k} - \sum_{a \in A} k(C^k_a \cdot F^k_a) - \\
& \quad \sum_{a \in A} k(C^k_a \cdot F^k_a \cdot CD_a) - \\
& \quad \sum_{a \in A} k P^k_a \sum_{w \in W} \sum_{(p,k) \in PK_w} k \delta_{a,(p,k)} \cdot q^w_{p,k} \\
\text{s.t.} \quad & \sum_{w \in W} \sum_{(p,k) \in PK_w} k(\delta_{a,(p,k)} \cdot q^w_{p,k}) \leq S^k_a \cdot F^k_a & \forall a \in A; k \in K^{air} \\
& V_{C_n} \leq 1 & \forall n \in N^{air} \\
& LR^w_{p,k} \leq R^w_{p,k} \leq UR^w_{p,k} & \forall (p,k) \in PK_w; w \in W \\
& LF^k_a \leq F^k_a \leq UF^k_a & \forall k \in K; a \in A^k
\end{align*}$$ (10.1)
where

\[ X = \ldots, X^k, \ldots \in \Omega \]  
\[ X^k = (R^k, F^k) \in \Omega^k \]  
\[ R^k = \ldots, R^w_{p,k}, \ldots \]  
\[ F^k = \ldots, F^a, \ldots \]  

and \( \Omega = \prod_{k \in K} \Omega^k \), where \( \Omega^k = \{ X^k \} \) (10.2) and (10.5) hold is the feasible strategy set of carrier \( k \). \( C^k_a \) is determined by Equations (1)-(2) and \( c^k_a \) is the unit cost of congestion delay to carrier \( k \) on arc \( a \), approximated by the per unit time operating cost \( \left( \frac{c^k_a}{s^a} \right) \). \( \rho^k_a \) is the constant marginal per passenger costs on arc \( a \) incurred by carrier \( k \). Constraint set (10.2) guarantees that passenger demand on each arc does not exceed the seating capacity offered by each carrier on that arc (i.e., carriers cannot sell tickets more than their capacity). Constraint set (10.3) restricts carriers’ decisions on service frequencies such that the capacity of each airport is not violated. In other words, (10.3) implies that the total number of aircraft of all carriers landing at an airport during a period of time should not be more than the airport’s capacity. Therefore, (10.3) is a joint constraint, resulting in a generalized Nash equilibrium problem (GNE). Finally, constraint sets (10.4)-(10.5) restrict the supply factors to predefined lower and upper bounds.

### 3.2. Demand: stochastic user equilibrium (SUE) with elastic demand

The two-level Nested Logit function defined in Equations (7)-(9) explains the interaction between travelers, who simultaneously decide on whether or not to travel and choose of route-carrier mix by observing the decisions of carriers such as fare and service frequency. However, one cannot directly use this demand function since road congestion delay, which is a key factor in the road users’ utilities, is itself dependent upon flow. More specifically, if we exclude road, we note that demand is a function of only fare and frequency \( q = f(R, F) \) chosen by the carriers, which are both observed by the followers. However, the congestion delay term in the road utility function is dependent upon the demand itself, which precludes direct utilization of the NL function. This difficulty is tackled by solving an equivalent problem to the NL function.

The equivalent nonlinear optimization problem for the stochastic user equilibrium problem with elastic OD demand is presented in (12.1)-(12.4). Equivalence between this optimization problem and the NL demand function in (7)-(9) is proved in Appendix 1.

\[
\begin{align*}
\min_{Q} z(Q) &= \sum_{w \in W} \sum_{(p,k) \in PK_w} q^w_{p,k} (\ln q^w_{p,k} - 1) - \sum_{w \in W} q^w_w (\ln q^w_w - 1) - \\
&\quad \frac{1}{g} \sum_{k \in \text{road}} \int_0^{v^k_{a,k}} v^k_{a,k}(\omega) d\omega - \frac{1}{g} \sum_{k \in \text{road}} V^k_{a,k} v^k_a + \\
&\quad \frac{1}{g} \sum_{w \in W} \int_0^{Q^{-1}} (y) dy \\
\text{s.t.} \quad (12.1)
\end{align*}
\]  
\[
\begin{align*}
\sum_{(p,k) \in PK_w} q^w_{p,k} &= q^w_w \quad \forall w \in W \quad (12.2) \\
q^w_{p,k} &\geq 0 \quad \forall (p,k) \in PK_w; w \in W \quad (12.3) \\
q^w_w &\geq 0 \quad \forall w \in W \quad (12.4)
\end{align*}
\]

where \( Q = (\ldots, q^w_{p,k}, \ldots, q^w_w, \ldots) \) \( \in \Phi \), \( \Phi \) is the feasible set governed by constraints (12.2)-(12.4), \( Q^{-1}(q^w_w) \) is the inverse demand function associated with OD pair \( w \), and \( V^k_{a,k} \) refers to the utility of a passenger by traveling along arc \( a \) by carrier \( k \), which can be considered as the building blocks of \( V^w_{p,k} \) in (5), i.e., \( V^w_{p,k} = \sum_{a \in A^k} V^k_{a,k} \delta_{a,(p,k)} \). The dual variables (Lagrange multipliers) of the constraints are shown in parentheses. Constraint set (12.2) implies flow conservation for each OD pair, and nonnegative flow constraints are considered in (12.3)-(12.4).
The feasible set \( \Phi \), composed of linear and non-negativity constraints, is non-empty, closed, and convex. In addition, the saturated demand provides upper bounds for OD pair demands. Therefore, \( \Phi \) is compact. As a result, at least one solution for the above nonlinear optimization problem exists.

According to the minimum principle (Facchinei and Pang, 2000), the necessary optimality conditions for this optimization problem can be expressed as an equivalent variational inequality (VI) problem in (13.1)-(13.2), which aims at finding a feasible flow vector \( Q \) such that the following inequality holds true.

\[
(Q - Q^*)^T H(Q^*) \geq 0 \quad \forall Q \in \Phi
\]

\[
Q = \begin{bmatrix}
\vdots \\
q_{p,k}^w \\
\vdots \\
q_w
\end{bmatrix} \in \Phi \\
H(Q) = \nabla_q z(Q) = \begin{bmatrix}
\vdots \\
-\frac{1}{\theta} V_{p,k}^w + \ln(q_{p,k}^w) \\
\vdots \\
\frac{1}{\theta} Q_w^{-1}(q_w) - \ln(q_w)
\end{bmatrix}
\]

### 3.3. Supply-demand-performance equilibrium

We have so far shown the competition between carriers who anticipate travelers’ decisions and also the interaction between travelers who observe carriers' decisions. In this section, we characterize multimodal transportation system equilibrium by formulating the multi-leader-follower game as a novel mathematical problem.

If we exclude road, the interaction between travelers can be explained directly by using the NL demand function introduced in Equations (7)-(9) rather than the equivalent VI problem in (13). This NL function could be then plugged into the profit maximization problem of each carrier in (10), thereby coming up with a single-level mathematical program for each carrier. Therefore, the multi-leader-follower game can be solved by solving a sequence of single-level mathematical problems. However, when road is included, each carrier will face a bi-level mathematical problem. More precisely, since each carrier’s bi-level problem consists in a VI problem in the lower-level, which represents user equilibrium conditions, it is in the class of the more general mathematical problems with equilibrium constraints (MPEC). The supply-demand-performance equilibrium is then computed by coupling all MPECs, as in Expressions (14.1)-(14.3), which will result in an equilibrium problem with equilibrium constraints (EPEC). EPECs have very recently garnered attention among mathematician, electrical engineers, and energy scientists (e.g., Ehrenmann, 2004; Lin, 2005; Steffensen and Bittner, 2014) and is still in its infancy. To the best of our knowledge, this is the first study in transportation science considering EPEC for solving a multi-leader-follower game.

\[
\text{Max } \pi_k^* (X^k; X^{-k}; Q) \quad \forall k \in K
\]

s.t.

\[
Q \in S(X)
\]

\[
(X, Q) \in \Omega \times \Phi
\]

where \( S(X) \) is the solution set of the \( VI(H(X, Q), \Phi(X)) \), which is parametrized by the observed carriers' fare and service frequency decisions.

### 4. Infrastructure investment decisions

Since investment decisions shift the current equilibrium of the transportation system, one should carefully consider the response of carriers and travelers by re-evaluating system equilibrium under the new system conditions. In this paper, we consider a central social planner (e.g., a government) who aims to foresee the welfare implications of its different investment policies for expanding the multimodal transportation network under its control.
As there are three groups of agents influenced by investment decisions (travelers, carriers, and the central planner), system welfare can be quantified as the travelers’ surplus plus carriers’ surplus (sum of profits of all carriers) minus the central planner’s investment cost. Considering $Y$ as the decision vector of the planner, system welfare ($SW$) is computed as a function of $Y$ in (15). Following (Adler et al., 2010; Adler et al., 2014; Small and Rosen, 1981; Train, 2003), the travelers’ surplus associated with the Logit model is defined as the maximum expected utility quantified in monetary terms and formulated as the first term in (15).

$$SW(Y) = \sum_{w \in W} \frac{\bar{q}_w \gamma \rho w \ln \left( \sum_{(i,j) \in PK} \exp \left( \frac{1}{\rho} V_{ij} \right) \right)}{\text{Travelers' surplus}} + \sum_{k \in K} \pi_k + \sum_{a \in A} \left[ \eta_a y_a L_a + \eta_n y_n L_n \right]$$

where $Y = (..., y_a, ..., y_n, ...)$ is the planner’s decision vector, within which $y_a(y_n)$ is the continuous decision variable for capacity expansion on arc $a$ (node $n$), $\rho_a$ is the unit costs of capacity expansion, and $AM_a$ is the long-run maintenance cost of arc $a$ incurred by the planner.

5. Solution algorithm

The equilibrium solutions for fares and frequencies of all carriers under a specific investment scenario can be determined by solving the EPEC in Expressions (14.1)-(14.3). We solve this EPEC by an algorithm that combines the Gauss-Seidel diagonalization method and the relaxation (regularization) method. The basic idea is to solve MPECs in (14.1)-(14.3) separately and sequentially using the relaxation method, holding the decision variables of other carriers fixed in turn until the sequence converges. The step-by-step procedure is as follows:

Step 1: Initialization. Choose initial fare and frequency vectors for all carriers.
Step 2: Determination of flow patterns. Determine the resultant equilibrium passenger flows using the VI problem in (13.1)-(13.2).
Step 3: Diagonalization. Solve the MPEC problem separately and sequentially for all carriers. Each MPEC is solved by, first, converting the MPEC into an equivalent single-level non-linear program (NLP) by using the relaxation (regularization) method, which relaxes the complementarity constraints of the VI problem, and then using the reduced gradient method to solve the equivalent NLP.
Step 4: Updating. Update fare and frequency vectors.
Step 5: Convergence check.

6. Numerical analysis

The proposed EPEC in (14) is implemented on the test network depicted in Figure 3, which represents one layer of the simple multimodal transportation network between three cities in a region. These cities are connected to each other by direct road and air links with the link numbers shown on the figure. Without loss of generality, each route on this test network is assumed to be composed of one link with the route numbers the same as arc numbers.

![Figure 3. Test network](image-url)
Network elements’ properties such as lengths of arcs in air/road networks, capacities of arrival airports in terms of arrival flights per hour, number of lanes on each road arc, and arc capacity in terms of passenger car per hour are assumed as given inputs reported in Table 2. Arc lengths are analogous to the network between Chicago, Detroit, and Minneapolis.

Table 2. Network elements’ properties

<table>
<thead>
<tr>
<th>arc/route</th>
<th>Air network</th>
<th>Road network</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Length (mile)</td>
<td>Arrival airport capacity (flights/hour)</td>
</tr>
<tr>
<td>1</td>
<td>339</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>339</td>
<td>40</td>
</tr>
<tr>
<td>3</td>
<td>232</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>232</td>
<td>40</td>
</tr>
<tr>
<td>5</td>
<td>528</td>
<td>40</td>
</tr>
<tr>
<td>6</td>
<td>528</td>
<td>40</td>
</tr>
</tbody>
</table>

Model parameters are shown in Table 3. Regarding Logit parameters, $\gamma_R$ and $\gamma_{ST}$ are extracted from the empirical study by (Hsiao and Hansen, 2011), which suggests a value of time equal to 19.4 dollars per hour for each traveler. Similar to (Li et al., 2010), it is assumed that passengers are more responsive to schedule delay and congestion delay than scheduled travel time by 30 percent. The parameters of the airport arrival delay function are borrowed from (Zou, 2012). It is also assumed that the reasonable upper (lower) bounds on fare and service frequency are 500 (0) and 12 (0), respectively. The marginal passenger cost, which is used in computing passenger-related costs incurred by carriers, is assumed to be 20 dollars per passenger, according to (Li et al., 2010).

Table 3. Model parameters

<table>
<thead>
<tr>
<th>Logit parameters</th>
<th>Delay parameters</th>
<th>Supply bounds</th>
<th>Marginal passenger cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_R$</td>
<td>$\delta_0$</td>
<td>LR</td>
<td>$\rho$</td>
</tr>
<tr>
<td>$\gamma_{ST}$</td>
<td>$\delta_1$</td>
<td>UR</td>
<td>20</td>
</tr>
<tr>
<td>$\gamma_{SD}$</td>
<td>$\delta_2$</td>
<td>LF</td>
<td>0</td>
</tr>
<tr>
<td>$\gamma_{CD}$</td>
<td>$\alpha$</td>
<td>UF</td>
<td>12</td>
</tr>
<tr>
<td>$\theta$</td>
<td>$\beta$</td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

The EPEC in (14.1)-(14.3) is written in the General Algebraic Modeling System (GAMS), and the solution algorithm is implemented in GAMS. The relaxation step is automatically conducted in the NLPEC solver, the result of which is then passed on to the NLP solver CONOPT that uses a reduced gradient algorithm.

Assuming a maximum potential OD trip equal to 80,000 passengers per day for all OD pairs, the results of the multimodal system equilibrium when two airlines are competing for travelers while road being another travel option are presented in Table 4. According to Table 4, carriers will charge higher fares and will provide more frequent services between more distant markets. It is intuitive as road is a more competitive travel mode in short-haul trips, which is correctly reflected in the larger route flows on shorter road links.

Table 4. Equilibrium fares, service frequencies, and route-carrier flows

<table>
<thead>
<tr>
<th>arc/route</th>
<th>Fare ($)</th>
<th>Frequency (flights/day)</th>
<th>route-carrier mix flow (pax/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>carrier 1</td>
<td>carrier 2</td>
<td>carrier 1</td>
</tr>
<tr>
<td>1</td>
<td>88.2</td>
<td>88.2</td>
<td>8.7</td>
</tr>
<tr>
<td>2</td>
<td>88.2</td>
<td>88.2</td>
<td>8.7</td>
</tr>
<tr>
<td>3</td>
<td>74.4</td>
<td>74.4</td>
<td>5.7</td>
</tr>
<tr>
<td>4</td>
<td>74.4</td>
<td>74.4</td>
<td>5.7</td>
</tr>
<tr>
<td>5</td>
<td>124.2</td>
<td>124.2</td>
<td>12.0</td>
</tr>
<tr>
<td>6</td>
<td>124.2</td>
<td>124.2</td>
<td>12.0</td>
</tr>
</tbody>
</table>
Table 5 shows the realized OD demand, which reveals the effect of market characteristics (e.g., distance) on the trip rate. Carriers’ profit, revenue, and cost breakdown are detailed in Table 6. Results indicate that each carrier raises approximately $670,000 per day in revenue, almost 30 percent of which is net profit. Nearly two-third of each carrier’s costs are associated with aircraft-related expenses (i.e., operating costs), while congestion delay costs and passenger-related costs contribute almost 5 percent and 28 percent to the total cost, respectively.

<table>
<thead>
<tr>
<th>Table 5. Equilibrium realized OD demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Table 6. Carriers’ profit, revenue, and cost breakdowns

<table>
<thead>
<tr>
<th>(units: $/day)</th>
<th>carrier 1</th>
<th>carrier 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>200,202</td>
<td>200,191</td>
</tr>
<tr>
<td>Revenue</td>
<td>669,814</td>
<td>669,261</td>
</tr>
<tr>
<td>Operating cost</td>
<td>314,083</td>
<td>313,742</td>
</tr>
<tr>
<td>Congestion delay cost</td>
<td>23,677</td>
<td>23,649</td>
</tr>
<tr>
<td>Passenger-related cost</td>
<td>131,852</td>
<td>131,680</td>
</tr>
</tbody>
</table>

Several sensitivity analyses can also be conducted. For example, if the saturated OD demand is halved to 40,000 passengers per day, carriers will provide less frequent services while charging lower fares, which results in nearly 70 percent decrease in a carrier’s profit. As another example, if we intensify the competition between carriers by entering two more airlines into all markets (i.e., four competing airlines in total), carriers will decrease their fares while almost keeping the same service frequencies, which will result in more passengers attracted to air travel than road. Besides, more intense competition translates into loss of profit for carriers by 20 percent compared to the case when two carriers were competing.

7. Conclusion

This paper proposes a novel equilibrium-based benefit assessment framework for infrastructure investment decisions. Compared to the previous studies that performed partial equilibrium analyses by considering either private (road) or public (air) transportation systems separately, this paper conducts a general equilibrium analysis where private and public transportation are modeled in one single framework. This novel framework explicitly captures decisions of all system components by modeling fare and service frequency competition between airlines, and travelers’ joint mode-route choices. The interaction between system components results in a multi-leader-follower game, which is formulated as an equilibrium problem with equilibrium constraints (EPEC). The proposed EPEC is solved by combining Gauss-Seidel diagonalization method and relaxation method and is implemented on a test network. The next step, which is currently in process, is to implement this framework on the real size intercity transportation network of the U.S. Midwest region.

Appendix 1. Proof of equivalence between the optimization problem (12) and the NL demand model

Because the feasible region of the optimization problem (12) is polyhedron, it has an equivalent Karush-Kuhn-Tucker (KKT) system, written as:

\[
\ln q_{p,k}^w - \frac{1}{\theta} \sum \int_{k=road} \frac{\partial f_a(v_a(x),\omega)\mu_a}{\partial q_a} \frac{\partial v_a}{\partial q_{p,k}} - \frac{1}{\theta} \sum V_{a,k} \frac{\partial v_a}{\partial q_{p,k}} - \mu^w - \lambda^w = 0 \quad \forall (p,k) \in PK_w, w \in W \tag{A.1.1}
\]

\[-\ln q_w + \frac{1}{\theta} \int \frac{\partial f(w)}{\partial q_w} dy - \mu^w - \lambda^w = 0 \quad \forall w \in W \tag{A.1.2}
\]

\[\sum_{(p,k) \in PK_w} q_{p,k}^w = q_w \quad \forall w \in W \tag{A.1.3}
\]
0 \leq \lambda_{p,k}^w \perp q_{p,k}^w \geq 0 \quad \forall (p,k) \in PK_w; w \in W \quad (A.1.4)

0 \leq \lambda^w \perp q_w \geq 0 \quad \forall w \in W \quad (A.1.5)

Note that Equations (A.1.1) and (A.1.2) ensure $q_{p,k}^w > 0$ and $q_w > 0, \forall (p,k) \in PK_w, w \in W$. The complementarity conditions in (A.1.4) and (A.1.5) immediately translate into $\lambda_{p,k}^w = 0$ and $\lambda^w = 0$, respectively.

Considering $\frac{\partial}{\partial q_{p,k}} V_{a,k}(\omega) = V_{a,k}(v_a)$ and $\frac{\partial v_{a,p}}{\partial q_{p,k}} = \delta_{a,(p,k)}$, which is suggested by (4.1), Equation (A.1.1) for each route-carrier combination can be simplified to

$$\ln q_{p,k}^w - \frac{1}{\theta} V_{p,k}^w + \mu^w = 0 \quad (A.2)$$

Simple algebra leads to

$$q_{p,k}^w = \exp \left( \frac{1}{\theta} V_{p,k}^w - \mu^w \right) \quad (A.3)$$

Summing over all route-carrier combinations in $PK_w$ yields (to avoid confusion, we use $(i,j)$ to denote an arbitrary route-carrier combination within $PK_w$):

$$\sum_{(i,j) \in PK_w} q_{i,j}^w = \exp (-\mu^w) \sum_{(i,j) \in PK_w} \exp \left( \frac{1}{\theta} V_{i,j}^w \right) \quad (A.4)$$

Note that the above equation also equals $q_w$, which results in a closed-form expression for $\mu^w$:

$$\mu^w = -\ln \left[ \frac{q_w}{\sum_{(i,j) \in PK_w} \exp \left( \frac{1}{\theta} V_{i,j}^w \right)} \right] \quad (A.5)$$

Plugging (A.5) back into (A.3) yields the route-carrier choice model for a given OD pair $w$, which has a simple Multinomial Logit form.

$$q_{p,k}^w = q_w \cdot \frac{\exp \left( \frac{1}{\theta} V_{p,k}^w \right)}{\sum_{(i,j) \in PK_w} \exp \left( \frac{1}{\theta} V_{i,j}^w \right)} \quad (A.6)$$

Recalling $\frac{\partial}{\partial q_w} Q_w^{-1}(q_w) dy = Q_w^{-1}(q_w)$, for each OD pair Equation (A.1.2) implies

$$-\ln q_w + \frac{1}{\theta} Q_w^{-1}(q_w) - \mu^w = 0 \quad (A.7)$$

Simple algebra results in

$$q_w = \exp \left( \frac{1}{\theta} Q_w^{-1}(q_w) \right) \cdot \exp(-\mu^w) \quad (A.8)$$

After substituting $\mu^w$ by the right hand side of equation (A.5), (A.8) can be alternatively expressed as:

$$q_w = \exp \left( \frac{1}{\theta} Q_w^{-1}(q_w) \right) \frac{q_w}{\sum_{(i,j) \in PK_w} \exp \left( \frac{1}{\theta} V_{i,j}^w \right)} \quad (A.9)$$

that immediately leads to
\[ q_w = Q_w \left( \theta \ln \left[ \sum_{(i,j) \in PK_w} \exp \left( \frac{1}{\theta} v^w_{i,j} \right) \right] \right) = Q_w(\theta l_w) \]  

(A.10)

Combining expressions (A.6) and (A.10) gives the two-level Nested Logit model specified in (7)-(9).

References
22. U.S. Bureau of Transportation Statistics. (October 2014) Table 3-17: Average cost of owning and operating an automobile.